Showing Ads to the Wrong Consumers: Strategic Ad Platform Inefficiency in Online Targeted Advertising

Note: 3rd Paper

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Abstract

I find that an ad platform has an incentive to induce lower product prices. Consumers pay a search cost when clicking on an ad. To induce consumer clicking, an ad platform adopts a targeting strategy that induces the merchant to lower its price. This involves showing the ad to some consumers who it rationally expects not to buy the product and not showing the same ad to other consumers who it would rationally expect to buy the product.

JEL Codes: M37; D83; L19; C78

KEYWORDS: online targeted pay-per-click advertising; online advertising platforms; strategic inefficiency; search; match

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1 Introduction

Whenever consumers search, communicate, buy, and surf online, they are bombarded with pop-up ads, banner ads, and sponsored-link ads. Online advertising has become an integral part of consumers' cyber-lives and in many cases an integral part of their real lives. Computing has displaced the radio as our second most time-consuming media outlet (CMD, et al., 2009).[1] At the same time, online advertising has become the third largest advertising market in the United States (PricewaterhouseCoopers, 2010). Yet, online advertising is different from traditional advertising through a much higher prevalence of "targeted advertising" and a high prevalence of "pay-per-click" (PPC) pricing, which is a method of charging for advertisements that is unique to online advertising. In this paper I present a model where an online ad platform "targets" advertisements to maximize its profit under pay-per-click pricing. I find that an online ad platform would not necessarily advertise in the same way that the merchant would, especially when consumers need to be induced to click on the advertisement.

Targeted advertising is when different ads are shown to different consumers based on tastes, locations, or demographics. By advertising for Geno's Cheesesteaks in The Philadelphia Inquirer or for Meow Mix in Cat Fancy, advertisers hope to increase the effectiveness of their ads. Yet targeted advertising is more prevalent and more precise on the internet, because Google, Amazon, Facebook, and other online advertising platforms are better able to personalize ads to fit consumer characteristics and therefore better able to induce consumers to click on their ads through more information about consumers and more precise computing technology (Bergemann and Bonatti, 2011). Google tracks what content we view and what searches we perform through temporary internet files and uses it to show us personalized ads.[2] Amazon uses our shopping histories to recommend products and services to us.[3] Facebook uses our profiles to personalize ads by age, gender, keywords, education, workplace, relationship status, relationship interests, and languages.[4]

[1] Television remains the most time-consuming media outlet.

[2] See `http://www.google.com/intl/en\_us/ads/ads\_1.html/` for details.

[3] See `http://www.amazon.com/gp/seller-account/mm-summary-page.html/ref=gw\_m\\\_b\_awus?ie=UTF8\&ld=AZAdvertiseMakeM\&topic=200260730` for details.

[4] See `http://www.facebook.com/advertising/` for details

Although initially online ad platforms charged a merchant per thousand viewers shown its ad, today most online ad platforms use *pay-per-click* (PPC) pricing.[5] Here a merchant pays for his ad based on the number of consumers who click on his ad or visit his site instead of based on the number of consumers that see his ad or buy his product. This pricing system is practically nonexistent in other forms of advertising like newsprint or television, because there is no easy, verifiable way to determine which ad should get credit for a consumer who visits the retailer. Instead newsprint and television advertisers pay based on the expected sales the ad will generate or the number of people who see the ad.

Some previous literature has examined targeted advertising (see for example: Athey and Gans, 2010; Bergemann and Bonatti, 2011), pay-per-click advertising (Agarwal et al., 2009) and the role of advertising platforms (Ghose and Yang, 2009). Yet only this paper and the parallel research of de Cornière (2011) analyze targeted pay-per-click advertising in a market with an ad platform. He primarilly explores the case where the merchants choose how to target their advertisements. A merchant chooses which consumers to show its ad based on consumer characteristics. I explore in this paper and he briefly considers in an extension the case where the ad platform chooses how to target advertisements. The ad platform chooses which consumers it shows an ad based on consumer characteristics. I get different results from his extension, because I allow the ad platform to have more control over which consumers see an ad. His targeting technology only allows the choice of a minimum reservation price. Consumers with lower valuations for the product would not see the ad, while consumers with higher valuations for the product would see the ad. In this paper, I show that the ad platform would not necessarily want to target ads in this way.

To do this I adapt a classic costly search model. Here, each consumer does not know how much he will value the product (their reservation price) until he clicks on an ad. A consumer will only click on the ad, if his expected benefit from doing so is more than the search cost, which is the opportunity cost or travel cost.[6]

In addition I let advertising be informative. Here, a consumer will not be able to buy the product or even click on the ad, unless the consumer is shown the ad. The ad platform can induce consumers to click

[5] This is sometimes called cost-per-click (CPC) pricing.

[6] This assumes there is no added utility from shopping.

by only showing the ad to a subset of consumers. The firm and the consumers know that if a consumer is shown the ad, they belong to this subset of consumers. This influences the firm's pricing decision and can increase the expected benefit from clicking on the ad.

Because ad platforms benefit by increasing the clicking on the ad, while merchants benefit by increasing its profits, their incentives generally are not aligned. I start by assuming that the pay-per-click price of an ad is exogenous. This assumption is common in the targeted advertising literature (see for example Anand and Shachar, 2009; Iyer et al., 2005; Johnson, 2013). Although special cases exist where the pay-per-click price is fixed,[7] in general how an ad platform targets its advertisements should influence what merchants are willing to pay for advertising. Therefore I then relax this assumption by allowing the ad platform to make a take-it-or-leave-it offer to the merchant when choosing how to target the ad.

In the first period of my game, each consumer draws his reservation price r for the merchant's product. This reservation price is not observed by the consumer, because the consumer does not know the merchant's product's characteristics, and it is not observed by the merchant, because the merchant does not know the consumer's characteristics. Only the advertising platform knows both, so only the advertising platform learns the consumer's reservation price r. In the second period of the game, the advertising platform chooses how the ad is targeted by choosing the proportion of consumers with each reservation price that is shown the ad; this proportion is observed by all agents. In the third period of the game, simultaneously the firm chooses its single price p, and each consumer who is shown the ad decides whether or not to click on it. Then all the consumers who clicked on the ad observe their reservation price r and the product price p. Then in the last period of the game, each consumer who clicked on the ad decides whether or not to purchase the product. Then the game ends with all agents recieving their payoffs.

The consumers who did not click on the ad get a payoff of zero, whether or not they were shown the ad. The consumers who clicked on the ad pay an ad annoyance cost b. I interpret the search cost as the forgone

[7]For example, many online advertising platforms, including Google, provide a minimum pay-per-click price. A fixed pay-per-click price would occur when only one advertiser is bidding on a keyword. See https://adwords.google.com/select/KeywordToolExternal for details.

opportunities and energy that the consumer gives up to review the product details.[8] If a consumer buys the product, then he gets payoff $r - p$ in addition to paying b. The firm gets a payoff equal to its price, p, for every consumer who buys the product while paying a fixed pay-per-click price c for every consumer who clicks on his ad. The payoff to the advertising platform is the advertising revenue from the firm.

When an online advertising platform's choice of to whom it shows an ad influences the number of clicks and not the pay-per-click price of an ad, then the platform will not show the ad to *some* consumers that it would rationally expect to buy the product in order to change the shape of the demand curve, inducing lower prices. This increases the expected benefit from clicking, leading to more clicking. For example, Google might show an ad for a Gershwin album to a rap-music-loving teenager and not to a music professor. And Amazon might show an ad for a book on game theory to a garage mechanic and not show the same ad to me. This shows the importance that pay-per-click pricing plays on how an ad platform targets its advertisements.

2 Literature Review

My model is similar to the Bertrand-Chamberlin-Diamond search models with a one-sided market found in Wolinsky (1986) and Anderson and Renault (1999), and most similar to de Cornière (2011) and Anderson and Renault (2006).

In de Cornière (2011), Anderson and Renault (2006), and this paper if consumers knew their reservation prices before clicking on the ad then we would face the classic Diamond (1971)'s Paradox. Yet limited information about a product in the ad, through ad content (Anderson and Renault, 2006) or through the consumer's knowledge that the ad was targeted toward them (de Cornière, 2011, and my model) induces clicking while avoiding the Diamond (1971) Paradox.

In Anderson and Renault (2006), a merchant encourages some consumers to click on its ad or travel to its store through the content of its advertising. Their advertising content acts similarly to targeted advertising in my model, signaling a consumer whether or not to click on an ad. The key difference between our models

[8]The consumer was not already looking for a product when the ad popped up. I am not modeling the decision to surf the net. I am modeling the decision to click on an ad. The decision to surf is exogenous.

is that I include an advertising platform and pay-per-click pricing.

Like de Cornière (2011), I find that an advertising platform would not target-advertise in the same way that a merchant would. His targeting technology only allows the choice of a minimum reservation price. Consumers with lower valuations for the product would not see the ad, while consumers with higher valuations for the product would see the ad. He finds that the ad platform would not set the same minimum reservation price that the merchant would, sometimes higher and sometimes lower. I relax this targeting technology by allowing the ad platform to choose (with some reasonable constraints) the proportion of consumers that it shows the ad to for each reservation price. I show that for high enough search costs the ad platform would show the ad to some consumers who it rationally expects not to buy the product and not showing the same ad to other consumers who it would rationally expect to buy the product.

In addition, we have two key differences in our models: 1) de Cornière (2011) is a repeated search game while my model is not repeated, and 2) he assumes that consumers are already induced to click on the first ad. This causes him to find in the case where the ad platform chooses how to target the advertisements that targeted advertising induces higher product prices, while I find that they lead to lower product prices. In his paper a consumer chooses when to stop looking at ads (through a repeated costly search game) while in this paper a consumer chooses whether or not to look at the first ad (and pay the first search cost). For example, it would be more reasonable to use his model when consumers are searching for a product through a list of ads on Amazon. While it would be more reasonable to use my model when consumers are deciding whether to click on a banner ad on a webpage.

2.1 The Effect of Targeted Advertising on Product Price

The prevailing reasoning in the informative targeted advertising literature is targeted advertising induces higher product prices by reducing price competition (see for example: Iyer et al., 2005).[9] In this paper, I show that an ad platform will targeted advertise in a way that lowers prices to discourage ad avoidance. De

[9]Willmore (2008) also argues that this was the case in persuasive targeted advertising. When targeted advertising is treated as a signal of heterogeneous product characteristics, Anand and Shachar (2009) argues that targeted advertising can induce lower product prices because a false signaling merchant can use lower prices to compensate consumers.

Cornière (2011) argues that an platform would strategically target the ad to induce higher product prices further so the ad platform can charge more for advertising. I get a different result than de Cornière (2011), because in de Cornière (2011) a consumer chooses when to stop looking at ads while in this paper a consumer chooses whether or not to look at the first ad.

Under a product monopoly, Esteban et al. (2001) argues that targeted advertising sometimes induces higher product prices because advertising may effectively become a fixed cost instead of a marginal cost. I get a different result from Esteban et al. (2001), because I show that under pay-per-click targeted advertising the merchant would still treat the cost of advertising like a fixed cost. Under a product monopoly with endogenous product quality, Esteban et al. (2006) argues that targeted advertising sometimes induces higher product prices and sometimes induces lower product prices depending on which consumers are more willing to pay for additional quality: those with higher valuations for the product or those with lower valuations for the product. My results are not comparable with his because I do not endogenize product quality.

When costs of finding a consumer willing to buy the product in two different market segments are asymmetric, Galeotti and Moraga-González (2004) argues that targeted advertising induces higher product prices in the more expensive segment and lower prices in the less expensive segment. In my model, in a sense, the ad platform is making the market into a more expensive segment. Fewer consumers who see the ad will be willing to buy the product. Yet the ad platform does so in a way that induces lower product prices, not higher. I get a different result than Galeotti and Moraga-González (2004), because in my model, the ad platform is targeting the advertisement in a specific way to change the shape of the product's demand curve.

2.2 Ad Avoidance

In my model, I allow consumers to choose whether or not to click on an ad, therefore *avoiding* paying the search cost to learn about their reservation price for the product, learn the price of the product, and get a chance to buy the product. This choice is a choice of *advertisement avoidance*. Johnson (2013) studied the effect of targeted advertising with advertisement avoidance. In Johnson (2013), an individual merchant ignores the effect of its targeting strategy on consumer ad avoidance strategies, and the product price is exogenous. In my model, the ad platform is strategically playing off the rationally expected product price and the ad avoidance decision of the consumers to maximize its own profit.

3 The Model

Here I present a basic costly search model of a two-sided market for online targeted advertising. In one side of the market, an online *merchant* buys the opportunity to offer its product to consumers from a single, monopolist online *advertisement platform* through informative advertising. In the other side of the market, there is a unit mass of consumers whom the ad platform could choose to show the ad. Those consumers who are shown the ad choose whether to suffer ad annoyance in order to have the opportunity of buying the merchant's product. The advertising platform chooses which consumers see the ad in order to maximize the rational expectation of its ad revenue.

I present the model in the order of the timing of the game in order to avoid confusion about who knows what when. I will discuss each agent's objectives when I discuss their choices.

Phase 0/Setup, the *reservation price allocation phase*: The merchant's product is summarized by a characteristic x, which is a point drawn from a uniform distribution on the unit circle. The value of x is observed by the merchant and the platform, but not by the consumers. This assumption is motivated by the idea that only those agents who are familiar with the product know the value of x.[10]

Each consumer's taste is summarized by a characteristic y, which is a point drawn from a uniform distribution on the unit circle. The value of y is observed by the consumer and the platform, but not by the merchant and the other consumers. This assumption is motivated by the idea that only those agents who are familiar with the consumer's tastes know the value of y.[11] This consumer has a reservation price $r \equiv 2|x - y|$ for the product. Because all consumers equidistant from x would have identical tastes for the

[10]The ad platform could have observed some of the merchant's past sales, or the ad platform could have examined the merchant's website itself.

[11]The ad platform could have observed some of the consumer's past purchases, or the consumer could have told the ad platform its tastes through online surveys.

product, I index each consumer by his reservation price r. Note only the ad platform knows both x and y, therefore only the ad platform knows r for each consumer. At this point, the merchant and the consumers only have the distributional knowledge that $r \sim i.i.d. U[0,1]$.

Phase 1, the *targeting decision phase*: the advertising platform informative advertises the merchant's product at a constant pay-per-click price c for a constant marginal cost normalized to zero. The advertising platform chooses the probability that a consumer with reservation price r sees the ad to maximize the advertising revenue (i.e. its own profit) $A \equiv cQ_c$. This choice is represented by the function $f : [0,1] \to [0,1]$: a consumer with a reservation price r has $f(r)$ chance of seeing the ad and $1 - f(r)$ chance of not seeing the ad. A consumer who sees no ad does not know about the product, and therefore cannot purchase the product. A consumer understands that he has been chosen to receive an ad conveys information about his match quality. This is in fact the only information content of the ad. The ad platform chooses the value of f for every price $p \in [0,1]$ such that f is *nice* . A function f is *nice* if it satisfies functional form conditions C1-5 in Appendix B; these conditions amount to assuming that f is not too discontinuous.

In order for targeted advertising to exist, I assume that the advertising platform's choice of targeting strategy f is publicly observed. This is to disallow a credibility issue, where the advertising platform tells the merchant and consumers it is using one targeting strategy while it is really using another.[12]

To simplify my notation I define the variable ω^i as one if consumer i is shown the ad and zero if consumer i is not shown the ad.[13]

Also to keep my model simple, I assume that the ad reveals no information about the product to the consumer. This is similar to most of the advertising literature.

[12]If the choice of f were not observed, any equilibrium would involve showing the ad to all consumers. You can see this if you suppose that the ad platform assumes that consumers would believe the strategy it announced. Then it would be optimal for the ad platform to choose to show the ad to all consumers and announce another strategy to try to fool the consumers. The consumers would rationally expect the ad platform to lie and show the ad to all consumers, so no equilibrium with $f(r) < 1$ for some $r \in [0,1]$ could exist.

[13]Note the ad platform is choosing f and not ω. Therefore all consumers with the same reservation price have the same chance of seeing the ad. This rules out equilibria where different consumers with the same r have different expectations from clicking on the same ad.

Phase 2, the *clicking and pricing decision phase*: Here the merchant chooses its single price p, and each consumer who is shown the ad chooses whether or not to click, simultaneously and independently.[14]

The merchant sells a single good over the Internet and faces a constant marginal cost normalized to zero. It chooses the price p of its product to maximize its expectation of its profit $\Pi \equiv pQ - cQ_c$, where Q is the quantity of consumers that buy its product, c is the pay-per-click cost of its advertisement, and Q_c is the quantity of consumers that click on its ad.

Each consumer r decides whether to click on the ad to maximize his expected utility $E[u(r)] \equiv (E[(r - p)\omega_b^r] - b)\omega_c^r$, where ω_b^r is one if consumer r buys the product (and otherwise is zero), b is the ad annoyance cost (or search cost) of clicking on the ad, and ω_c^r is one if consumer r clicks on the ad (and otherwise is zero). Note that consumer r is choosing ω_c^r. Further note that as consumer r has not learned the price of the product and his reservation price r so his future choice of ω_b^r may be at this point unknown.

I define the quantity of the quantity of consumers that click on the ad as $Q_c \equiv \int_0^1 \omega_c^r dr$.

Phase 3, the *price reassurance and reservation price revelation phase*: Each consumer r who clicked on the ad now sees the product price p and his reservation price r.[15]

Phase 4, the *sales phase*: Here the consumers who clicked on the ad choose whether or not to buy the product. Each consumer r who clicked on the ad chooses whether to purchase the product to maximize his utility $u(r) \equiv (r - p)\omega_b^r - b$. Note that consumer r is now choosing ω_b^r, so consumer r will buy the product (i.e. $\omega_b^r = 1$) if $r \geq p$. I assume that consumers buy when indifferent. If a consumer does not click on the ad, then he cannot buy the product, so his utility $u(r)$ is automatically zero.

I define the quantity of the product sold as $Q \equiv \int_0^1 \omega_b^r dr$. If $Q = 1$, then all consumers buy the product, and if $Q = 0$, then no consumers buy the product.

[14] The assumption that price and clicking is chosen after targeting advertising is motivated by the idea of a behemoth ad platform catering to many different markets. In addition it is motivated by the theoretical concept that you need to give the ad platform incentive not to show the ad to every consumer, so consumers would have to choose to click after the ad platform commits and announces its targeting strategy.

[15] In equilibrium, the product price that the consumers rationally expect will be the actual price. Hence, the revelation of the price can be seen as a reassurance of the price.

END, *Profits*: Here all the agents get their payoffs.

4 Each Consumer's Clicking Decision

In this section, I will start to recursively solve for a subgame perfect Nash Equilibrium. In the last section, I found that a consumer who has clicked on the ad will buy the product if his reservation price r is greater than or equal to the price p. Here, I solve for a consumer's decision whether to click on the ad once he has been shown the ad. In the next section, I will analyze the merchant's pricing decision. From these two sections I will be able to aggregate all the decisions made in Phase 2, the clicking and pricing decision phase. Then in Section 6, I will solve for the equilibrium by solving for the ad platform's decision of which consumers to advertise to. All lemmas and proofs are given in Appendix A.

To calculate his expected payoff, a consumer has two observed pieces of information: 1) the fact that he is shown the ad and 2) the advertising platform's targeting strategy. From this information he develops rational beliefs about the price p^*, which I will refer to as the rationally expected price, and a distribution of his possible reservation prices $f/\overline{F}$, where $\overline{F} \equiv \int_0^1 f(p)dp$. Weighting f by $1/\overline{F}$ makes a probability density function–the probability density function of a random reservation price from those consumers shown the ad.

Therefore if a consumer is shown the ad, then his expected benefit $b_c(p^*, f)$ from clicking on the ad is given by equation (1), where p^* is the consumer's rational exceptions or beliefs of the products price.

$$b_c \equiv E[max\{r - p^*, 0\}|\omega = 1] = \int_{p^*}^{1} (p - p^*)f(p)dp/\overline{F} \tag{1}$$

A consumer's expected payoff from clicking is his expected benefit b_c minus his search cost b. Thus I have the *consumer clicking condition* (CCC) given below.

Consumer Clicking Condition (CCC): A consumer who is shown the ad will click on the ad if and only if $b_c \geq b$.

This is an identical decision for all consumers that are shown the ad. Either all consumers who are shown the ad click or none click. Note that this means that if f satisfies C1-5 then the density function of consumers clicking f_c satisfies C1-5.

5 The Merchant's Pricing Decision

In this section, I solve for the merchant's pricing decision. This occurs simultaneously with the consumer's clicking decisions. In the following section, I will use the conditions found in both sections to solve for the advertising platform's targeting decision and solve the equilibrium.

The merchant chooses its price p^* to maximize its profit $(p-c)Q - cQ_c$. Because the merchant takes the advertising cost cQ_c as a constant sunk cost, the merchant chooses its price p^* to maximize its sales revenue pQ.

The merchant takes the density function f of consumers shown the ad and infers its demand function $Q(p)$. Suppose that the merchant believes that, conditional on being shown the ad, a consumer will click through with probability θ, regardless of r. Then he will anticipate demand of $Q(p^*) = \theta \int_{p^*}^{1} f(p)dp$. I have already established in section 4 that either all consumers who are shown the ad click or none click. Therefore the merchant believes that either $\theta = 0$ or 1. Therefore, the merchant sets prices according to the *merchant's profit maximizing condition* (MPMC) given below.[16]

Merchant's Profit Maximizing Condition (MPMC): The merchant will choose its price p^* such that $p^* = \arg\max_{p'} p' \int_{p'}^{1} f(p)dp$.

If f is continuous at p^*, then the merchant's profit maximizing condition is equivalent to the first-order-condition given in equation (2).

$$p^* f(p^*) = \int_{p^*}^{1} f(p)dp \tag{2}$$

[16] If $\theta = 0$, any price would maximize profit. Therefore I assume that the merchant will arbitrarily chose the price given by MPMC to rule out the uninteresting degenerate equilibrium where the merchant sets its price really high and none of the consumers click on the ad.

This equation is the standard Bertrand profit maximization first-order conditions for the demand curve $Q(p) = \int_p^1 f(r)dr$.

6 The Ad Platform's Targeting Decision

In this section, I solve for the advertising platform's choice of the proportion $f(p)$ of consumers with each reservation price $r = p$ to whom the ad is shown, given the consumer and merchant strategies discussed above. Because the pay-per-click price c has already been determined before the game, the advertising platform chooses its targeting strategy to maximize the quantity Q_c of consumers clicking. The advertising platform knows the consumer clicking condition (CCC) and the merchant's profit maximizing condition (MPMC), so the advertising platform is able to rationally expect who will click under any choice of f.

6.1 A Benchmark Result: Advertise to All Consumers

In this first subsection, I analyze when the advertising platform would show the ad to all consumers. Here, the function $f(p) = 1\{p \in [0,1]\}$, so by the profit maximizing condition, the merchant would set the standard monopoly price $p^m = 1/2$.

Anticipating this price outcome, consumers would click on the ad if the expected benefit b_c from clicking on the ad were greater than the search cost b. This happens when the consumer clicking condition (CCC) is met and simplifies to the condition given in equation (3).

$$b_1 \equiv \int_{p^m}^1 (p - p^m)dp = 1/8 \geq b \tag{3}$$

If this condition is met, then the advertising platform would want to advertise to everyone because doing so gets all consumers to click on the ad, yielding the maximum possible mass of consumers clicking, $Q_c = 1$.[17]

In addition, it would also be optimal for the ad platform to advertise to all consumers if the search cost b were so high that no consumer would ever click on the ad under any targeting strategy. In this case any targeting strategy is optimal.

[17] This is formalized in Lemma 1.

6.2 Advertise to Some Consumers

Next, I develop some preliminary results to examine the case when the advertising platform would show the ad to some but not all consumers. When $f(p) = 1\{p \in [0,1]\}$ would not induce consumers to click, while some other targeting strategy $\widetilde{f}$ would then the advertising platform would optimally show the ad to some but not all consumers.[18]

In this section I describe a functional form of f that the advertising platform would choose. I do so by finding a functional form of f that the advertising platform would weakly prefer. I show that given any targeting strategy $\widetilde{f}$ satisfying C1-5, there exists a targeting strategy f satisfying C1-5 and the functional form shown in Proposition 1.[19]

Proposition 1. *Given any targeting strategy $\widetilde{f}$ satisfying C1-5, there exists a targeting strategy f satisfying C1-5 and the following functional form condition that produces (weakly) more clicking:*

$$f(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 & \text{if } p \in [\underline{p}, p^*] \\ (p^*/p)^2 & \text{if } p \in (p^*, \widehat{p}] \\ 1 & \text{if } \widehat{p} \neq 1 \text{ and } p \in (\widehat{p}, 1] \end{cases}$$

where $0 \leq \underline{p} \leq p^* \leq \widehat{p} \leq 1$ and p^* satisfies (MPMC) for f.

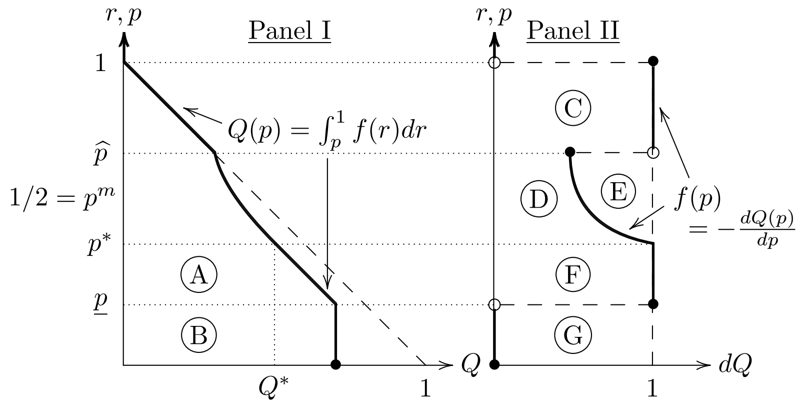
Figure 1 illustrates the targeting strategies shown in Proposition 1. The thick lines are various forms of the targeting strategy f in different spaces: I) the demand curve $Q(p) = \int_p^1 f(r)dr$ and II) the density function $f(p) = -\frac{dQ(p)}{dp}$. In panel I, I compare the demand curve $Q(p)$ to the function $1-p$ (i.e. the dashed line), the demand curve that the merchant would face if all consumers were shown and subsequently clicked on the ad. In panel II, I compare the density function f to the function $1\{p \in [0,1]\}$ to compare the density of consumers shown the ad to the density of all consumers at every reservation price.

The advertising platform uses the merchant's profit-maximizing condition to pick the merchant's rationally expected price p^*. Panel I illustrates this by having the merchant pick its price p^* to maximize $pQ(p)$,

[18] This is formalized in Lemma 2.

[19] I find this functional form piecemeal in lemmas 3, 4 and 5.

Figure 1: A Weakly Preferable Targeting Strategy



given by Ⓐ + Ⓑ. The merchant's first-order condition from equation (2) indicates how it would maximize its profit in Panel II. The merchant would pick p^* to equate the marginal loss $pf(p)$ from raising its price (i.e. Ⓕ + Ⓖ) with the marginal gain $\int_p^1 f(r)dr$ from raising its price (i.e. Ⓒ + Ⓓ).

The targeting strategy given in Figure 1 gives the merchant equal profit for setting any price between p^* and $\widehat{p}$. In Panel I, the box Ⓐ + Ⓑ of the merchant's before-advertising expense profit $pQ(p)$ is equal and maximized for any price $p \in [p^*, \widehat{p}]$. Thus the demand curve between prices p^* and $\widehat{p}$ is a demand curve of constant profit $Q(p) = (p^*/p)^2$. In Panel II, the box $pf(p)$ of marginal profit lost from raising the price (i.e. Ⓕ + Ⓖ) is equal to the region $Q(p) = \int_p^1 f(r)dr$ of marginal profit gained (i.e. Ⓒ + Ⓓ) for any price $p \in [p^*, \widehat{p}]$. This produces the curve between p^* and $\widehat{p}$.[20]

In the strategy depicted in Figure 1, the advertising platform is showing the ad to consumers $Q_c =$ Ⓒ + Ⓓ + Ⓕ and not to consumers Ⓔ + Ⓖ. This induces a rationally expected (weakly) lower product price, which makes the consumers more willing to click on the ad. In the next section I will show when it is optimal for an ad platform to show the ad to consumers who would not buy the product Ⓕ and not to consumers who would buy the product Ⓔ.

[20]Note that the condition that a merchant who is indifference between a set of prices would always pick the lowest price is playing a role here by making the merchant choose the price p^*. Without this condition, the ad platform could choose a strategy close to this strategy to guarantee the merchant choose the lowest price, by giving the merchant slightly more profit from choosing p^*. The problem with this is that for any close strategy, the ad platform can choose a strategy slightly closer to gain slightly more clicks.

6.3 Optimal Advertising

I now build upon the previous two subsections to find the model's equilibrium and the ad platform's optimal targeting strategy f for any search cost b and any pay-per-click price c. In Section 7 I will examine how the payoffs depend on b and c.

When $f(p) = 1\{p \in [0,1]\}$ would not induce consumers to click, while some other targeting strategy $\widetilde{f}$ would then the advertising platform would optimally show the ad to some but not all consumers.[21] The ad platform would choose to show the ad to just enough consumers so the search cost b equals the consumer's expected benefit from clicking on the ad b_c. If $b_c < b$, then the consumers would not be induced to click on the ad. If $b_c > b$, then the ad platform could increase its profits by increasing the amount of consumers it shows the ad.[22] Using this and Proposition 1, I find Proposition 2.

Proposition 2. *An optimal click-maximizing targeting strategy satisfies C1-5 and the following:*

(a) *when $b \leq b_1$: the advertising platform shows the ad to all consumers*

(b) *when $b_1 < b \leq b_2$: $\widehat{p} \in (p^m, p_1]$ and*

$$f(p) = \begin{cases} 1 & \text{if } p \in [0, p^*] \\ (p^*/p)^2 & \text{if } p \in (p^*, \widehat{p}] \\ 1 & \text{if } p \in (\widehat{p}, 1] \end{cases}$$

(c) *when $b_2 < b \leq b_3$:*

$$f(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 & \text{if } p \in [\underline{p}, p^*] \\ (p^*/p)^2 & \text{if } p \in (p^*, p_1] \\ 1 & \text{if } p \in (p_1, 1] \end{cases}$$

(d) *when $b_3 < b \leq b_4$: $\widehat{p} \in (p_1, p_2]$ and*

[21] This is formalized in Lemma 2.

[22] This is formalized in Lemma 2.

$$f(p) = \begin{cases} 0 & \textit{if } p < p^* \\ (p^*/p)^2 & \textit{if } p \in (p^*, \widehat{p}] \\ 1 & \textit{if } p \in (\widehat{p}, 1] \end{cases}$$

(e) *when $b > b_4$: no consumer would ever click on the ad so any f is optimal;*

where $0 < b_1 < b_2 < b_3 < b_4$, $0 < p^* \leq p^m < p_1 < p_2 < 1$, and p^* satisfies (MPMC) for f.

For low ad-annoyance costs, the platform focuses on excluding some potential inframarginal consumers by increasing $\widehat{p}$. While this does tend to make consumers more pessimistic about their r, conditional on seeing the ad, it also induces a lower p^* from the merchant. The latter effect is sufficiently strong so that not only do consumers become more willing to click, but they are more willing to click than if the ad platform had instead excluded consumers that would not be willing to buy the product by increasing $\underline{p}$. For somewhat higher ad-annoyance costs, the ad platform excludes additional potential consumers who it knows would not be willing to buy the product by increasing $\underline{p}$, because the additional potential inframarginal consumers that the ad platform would have to exclude would have higher higher reservation prices r. For even higher ad-annoyance costs, the platform would have to exclude even more potential inframarginal consumers by increasing $\widehat{p}$, because it would not be advertising to any potential consumers who it knows would not be willing to buy the product.

7 Examination of Equilibrium Payoffs

In this section I examine the payoffs of the equilibrium where: 1) the ad platform chooses the click-maximizing targeting strategy found in Proposition 2, 2) the merchant chooses its profit-maximizing price through equation (2) and 3) the consumers click when $b \leq b_4$. Table 1 summarizes my findings. I begin by examining a representative consumer's expected utility. Then I examine the ad platform's and the merchant's profits.

Table 1: Comparative Static Results of the Equilibrium Payoffs

<table>
<tr><th>Search Cost</th><th>Consumers</th><th>Ad Platform</th><th>Merchant
if $c > p^*$</th><th>Merchant
if $c < p^*$</th></tr>
<tr><td>$b < b_1$</td><td>$\frac{\partial E(u^i)}{\partial b} = -1$,
$\frac{\partial E(u^i)}{\partial c} = 0$</td><td>$\frac{\partial A}{\partial b} = 0$,
$\frac{\partial A}{\partial c} > 0$</td><td colspan="2">$\frac{\partial \Pi}{\partial b} = 0$, $\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b_1 < b < b_2$</td><td rowspan="4">$\frac{\partial E(u^i)}{\partial b} = 0$,
$\frac{\partial E(u^i)}{\partial c} = 0$</td><td rowspan="3">$\frac{\partial A}{\partial b} < 0$,
$\frac{\partial A}{\partial c} > 0$</td><td rowspan="3">$\frac{\partial \Pi}{\partial b} > 0$,
$\frac{\partial \Pi}{\partial c} < 0$</td><td>$\frac{\partial \Pi}{\partial b} < 0$, $\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b_2 < b < b_3$</td><td>$\frac{\partial \Pi}{\partial b} > 0$, $\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b_3 < b < b_4$</td><td>$\frac{\partial \Pi}{\partial b} < 0$, $\frac{\partial \Pi}{\partial c} < 0$</td></tr>
<tr><td>$b > b_4$</td><td>$\frac{\partial A}{\partial b} = \frac{\partial A}{\partial c} = 0$</td><td colspan="2">$\frac{\partial \Pi}{\partial b} = \frac{\partial \Pi}{\partial c} = 0$</td></tr>
</table>

7.1 Consumer Payoffs

By Lemma 6, if the search cost $b > b_1$, then a consumer's expected benefit b_c from clicking on the ad would be equal to b. Therefore his expected payoffs $E[u^i] \equiv b_c - b$ from clicking would be equal to zero. Therefore as long as $b > b_1$, $\partial E(u^i)/\partial b = 0$. Yet if $b < b_1$, then all consumers would choose click on the ad for any search cost b or for any merchant's marginal cost c of production. Also changing the pay-per-click price c does not change f or p^*, so $\partial E(u^i)/\partial c = 0$ for any b and c.

7.2 Ad Platform Profits

If the consumer search cost $b < b_1$, then the ad platforms profit A (i.e. the total ad revenue) would be equal to the pay-per-click price c times the total mass of consumers clicking Q_c, which equals one. Because both are exogenous, increasing or decreasing the consumer search cost b would not change the ad platform's profit A. If $b_1 < b < b_4$, then the ad platform's profit A would be equal to the mass of consumers shown the ad $\overline{F} \equiv \int_0^1 f(p)dp$ times the pay-per-click price c. Because $\overline{F}$ is decreasing in search cost b, so is the ad platform's profit A. Because $\overline{F}$ is uneffected by changing c, $\partial A/\partial c = Q_c = \overline{F} > 0$. Yet if $b > b_4$, then the ad platform cannot induce any consumers to click on the ad, so changing the search cost b or the pay-per-click price c would not affect the ad platform's profit A.

7.3 Merchant Profits

If the consumer search cost $b < b_1$, then the merchant sets the monopolist price p^m and its ad is shown to all consumers. Small changes in the consumer search cost do not discourage or encourage more consumers from clicking on the ad or buying the product, so $\partial \Pi/\partial b = 0$.

If $b_1 < b < b_4$, then by equation (2) I have $\partial \Pi/\partial \widehat{p} = 2(p^* - c) * \partial p^*/\partial \widehat{p}$. Therefore if the merchant faces a high enough pay-per-click price c, then the effect of changing the search cost b on the merchant's profit Π would be overwhelmed by the effect on the ad revenue A, so $\partial \Pi/\partial b > 0$. Yet if the merchant faces a low enough pay-per-click price c, then the effect of changing the search cost b on the merchant's profit Π would be overwhelmed by its profit from sales pQ. If $b_1 < b < b_2$ or $b_3 < b < b_4$, then a higher search cost b leads to a higher $\widehat{p}$, which leads to a lower price p^* and less profit Π. If $b_2 < b < b_3$, then a higher search cost b leads to a higher $\underline{p}$. The merchant's profit from sales pQ would not be affected and the merchant faces a lower advertisement cost A, so $\partial\Pi/\partial b > 0$.

If the consumer search cost $b > b_4$, then no consumer would ever click on the ad, so $\partial\Pi/\partial b$.

Also because c does not affect f, $\partial\Pi/\partial c = -\overline{F} = Q_c \geq 0$

8 Take-it-or-leave-it Offer Advertising

One criticism of my model is that I take the pay-per-click price of advertising as an exogenous constant c. In this section I explore when the effect of the targeting strategy influences the price of advertising. I analyze a simple adaptation to my model with an endogenous price of advertising.

8.1 The Take-it-or-leave-it Offer Model

Here I present my adaptation to my model to include endogenous pay-per-click pricing. Instead of a pay-per-click price c, the merchant chooses whether to accept or reject a take-it-or-leave-it offer from the advertisement platform that includes which consumers would be shown the ad and a fee ϕ for advertising.

Phase 0, the *reservation price allocation phase*: same as in Section 3.

Phase 1, the *targeting decision phase and take-it-or-leave-it offer phase*: Here the ad platform chooses f and makes a take-it-or-leave-it advertising offer ϕ to the merchant to maximize its expected profit A, which is ϕ when the merchant accepts the take-it-or-leave-it advertising offer and 0 otherwise. The advertisement platform chooses f as described in Section 3. f still must satisfy C1-5. Unlike Phase 1 in Section 3, consumers are not shown the ad in this phase. The ad platform is only committing to show the ad to the consumers specified by f if the merchant accepts the take-it-or-leave-it advertising offer.

Here the merchant and the consumers learn both the take-it-or-leave-it advertising offer ϕ and the targeting decision f. It is public knowledge.

After Phase 1 and Before Phase 2, the *offer acceptance phase*: Here the merchant chooses either to accept the take-it-or-leave-it offer (and set $A = \phi$) or to reject the take-it-or-leave-it offer (and set $A = 0$) to maximize its expectation of its profit Π which is $pQ - \phi$ when it accepts the take-it-or-leave-it offer and 0 otherwise.

If the merchant accepts this offer, then each consumer with a reservation price of $r = p$ is shown the ad with probability $f(p)$ and the merchant pays the ad platform ϕ for advertising (instead of paying $A = cQ_c$). If the merchant rejects the offer, then no consumers are shown the ad, and the ad platform and the merchant get a payoff of 0.

Phases 3-END: same as in Section 3, except A is ϕ when the merchant accepts the take-it-or-leave-it advertising offer and 0 otherwise.

8.2 The Take-it-or-leave-it Equilibrium

The merchant would accept any offer satisfying $\phi \leq p^* \int_{p^*}^1 f(p)dp$ and (CCC). The ad platform will make the largest take-it-or-leave-it offer that the merchant would be willing to accept, so the ad platform will set $\phi = p^* \int_{p^*}^1 f(p)dp$ and thus extract all the surplus from the merchant. The ad platform will choose its targeting strategy f to maximize the offer ϕ that the merchant would accept. Therefore the ad platform will choose a targeting strategy that maximizes the profit $p^* \int_{p^*}^1 f(p)dp$ of the merchant. One such targeting strategy is given in Proposition 3.[23]

Proposition 3. *An optimal merchant-profit-maximizing targeting strategy satisfies C1-5 and the following:*

(a) *when* $b \leq b_5$*:* $p^* = p^m$ *and*

$$f(p) = \begin{cases} 0 & \text{if } p < p^m \\ 1 & \text{if } p \in (p^m, 1] \end{cases}$$

(b) *when* $b_5 < b \leq b_4$*:* $\widehat{p} \in (p^m, p_2]$ *and*

$$f(p) = \begin{cases} 0 & \text{if } p < p^* \\ (p^*/p)^2 & \text{if } p \in (p^*, \widehat{p}] \\ 1 & \text{if } p \in (\widehat{p}, 1] \end{cases}$$

[23]Lemma 7 in Appendix A is used to prove Proposition 3. See Appendix A for a formal proof of both.

(c) *when* $b > b_4$*: no consumer would ever click on the ad so any* f *is optimal;*

where $0 < b_5 < b_4$*,* $0 < p^* \leq p^m < p_2 < 1$*, and* p^* *satisfies (MPMC) for* f*.*

The merchant gets no profit from advertising to consumers that it would never sell to. Therefore the ad platform has no reason to advertise to these consumers when trying to maximize the value of a take-it-or-leave-it offer ϕ, as seen in Proposition 3. Note that for low enough search costs $b \leq b_3$, the advertising platform would advertise to some consumers that would not buy the product to increase the number of clicks on the ad, as seen in Proposition 2. Therefore when $b \leq b_3$, take-it-or-leave-it offer advertising is more efficient than pay-per-click advertising with fixed pay-per-click prices.

Athey and Gans (2010) argued that when their is no private cost from advertising to consumers that would not buy the product, there is no need not to advertise to these consumers. When $b < b_1$, it is still optimal for the ad platform to show the ad to all consumers, because (CCC) would still be met and the merchant would get the same profit. Proposition 3 shows that when there is no private cost and no private gain from advertising to consumers that would not buy the product, there is no need to advertise to these consumers either.

Another interesting result of Proposition 3 is the ad platform's strategy when $b_5 \leq b \leq b_4$. Here the ad platform is not able to induce consumers to click on the ad with the targeting strategy $f(p) = 1\{p \in [p^m, 1]\}$, so the ad platform chooses a targeting strategy that will induce the merchant to charge a lower price. This is similar to the result in Anderson and Renault (2006). They explore endogenous advertising content in a costly search model. They find that when search cost are large enough, merchants will commit to lower prices in their advertisement content to induce consumers visit their store or click on their ad.

Also note that when search cost b satisfies $b \geq b_3$, then the targeting strategies chosen in Propositions 2 and 3 are identical. Therefore pay-per-click advertising with fixed pay-per-click prices is efficient when search costs b are large enough.

8.3 Which Model is More Realistic?

Exogenous pay-per-click pricing is not an unrealistic assumption. It means that the merchant has already committed to advertising at a pay-per-click price c when the ad platform chooses whom to show the ad. Also in industries with many merchants, it is reasonable to expect that merchants take the price of advertising as exogenous. Yet if there is little cost to drawing out complicated contracts between the ad platform and the merchant, then it is reasonable to assume the equilibrium is reflects the take-it-or-leave-it offer equilibrium. Therefore either model may be more appropriate depending on the industry.

9 Concluding Remarks

I found that when the platform is maximizing the number of clicks, it will not show the ad to some consumers that it would rationally expect to buy the product. And it will show an ad to some consumers that it would rationally expect not to buy the product. Targeting in this way changes the shape of the demand curve, inducing online merchants to lower their prices p^*. This increases the expected benefit from clicking b_c, leading to more clicking.

Future research in targeted advertising should look at the incentives of online advertising platforms. My model shows that the ad platform could show ads strategically inefficiently and de Cornière (2011) shows that an ad platform can over advertise. In Section 8, I showed how take-it-or-leave-it offer pricing can induce the ad platform to target its ads efficiently. Yet in reality, online advertising platforms sell a limited quantity of ad space through auctions. Future research should look at how online auctions and an advertising capacity constraint influence how ad platforms choose to target advertisements.

Also future research needs to test how competition between ad platforms and online merchants affects targeted advertising. Perhaps the kind of strategic inefficiency I found does not exists when multiple online ad platforms (say Yahoo and Google) compete to sell advertisement space to online merchants. Or perhaps the substitution between goods sold by online merchants induces ad platforms to differentiate the ads shown to different consumers.

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A Mathematical Proofs and Lemmas

Lemma 1. *If $b_1 = 1/8 \geq b$, then the advertising platform chooses to show the ad to all consumers.*

Proof of Lemma 1. If $f(p) = 1\{p \in [0,1]\}$ by (MPMC) the merchant would set the standard monopoly price $p^m = 1/2$. Therefore consumers would rationally expect the merchant to set the price $p^* = p^m = 1/2$. Therefore by (CCC), the consumers would click on the ad if $1/8 \geq b$. This gives the advertising platform its maximum possible ad revenue. C4 and C5 guarantee uniqueness by preventing removable discontinuities. □

Lemma 2. *If $b_1 < b$ and there exists a targeting strategy $\widetilde{f}$ satisfying C1-5 that would induce consumers to click, then the advertising platform will not advertise to every consumer.*

Proof of Lemma 2. The advertising platform strictly prefers targeting strategy $\widetilde{f}$ to $f(p) = 1\{p \in [0,1]\}$. Therefore f is not optimal. $\square$

Lemma 3. *Given any targeting strategy $\widetilde{f}$ satisfying C1-5, there exists a targeting strategy f satisfying C1-5 and the following functional form condition that produces (weakly) more clicking:*

$$f(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 & \text{if } \underline{p} \neq p^* \text{ and } p \in [\underline{p}, p^*] \end{cases}$$

where $0 \leq \underline{p} \leq p^* \leq 1$ and p^* satisfies (MPMC) for f.

Proof of Lemma 3. Let $\widetilde{p}$ be the price set by the merchant under $\widetilde{f}$. If $\widetilde{f}$ satisfies the functional form condition, then let $f = \widetilde{f}$ and $p^* = \widetilde{p}$. Otherwise $0 < \int_0^{\widetilde{p}} \widetilde{f}(p)dp < \widetilde{p}$, because $\widetilde{f}$ satisfies C1-3. If $\int_0^{\widetilde{p}} \widetilde{f}(p)dp = 0$ or $\widetilde{p}$ then $\widetilde{f}$ would satisfy the functional form condition. Let $\underline{p} \equiv \widetilde{p} - \int_0^{\widetilde{p}} \widetilde{f}(p)dp$. Let $f(p) \equiv 1\{\underline{p} \leq p \leq \widetilde{p}\} + \widetilde{f} * 1\{p > \widetilde{p}\}$. Because $f(p') = \widetilde{f}(p')\, \forall p' > \widetilde{p}$, we have that $p' \int_{p'}^1 f(p)dp = p' \int_{p'}^1 \widetilde{f}(p)dp\, \forall p' > \widetilde{p}$. Therefore by (MPMC), the merchant would prefer $\widetilde{p}$ to all prices $p' > \widetilde{p}$ under the targeting strategy $\widetilde{f}$. If the merchant would sets price $p^* = \widetilde{p}$ under the targeting strategy f, then f satisfies the functional form condition. If the merchant would set its price $p^* < \widetilde{p}$ under the targeting strategy f, then f adds more slackness to the consumer clicking condition because:

$$\begin{aligned} \int_{p^*}^1 (p - p^*) f(p)dp &= \int_{\widetilde{p}}^1 (p - \widetilde{p})\widetilde{f}(p)dp + (\widetilde{p} - p^*) \int_{\widetilde{p}}^1 \widetilde{f}(p)dp + \int_{p^*}^{\widetilde{p}} (p - p^*)dp \\ &> \int_{\widetilde{p}}^1 (p - \widetilde{p})\widetilde{f}(p)dp \end{aligned} \tag{4}$$

Therefore if consumers would click on the ad under the targeting strategy $\widetilde{f}$, then consumers would click on the ad under targeting strategy f. $\square$

Lemma 4. *Given any targeting strategy $\widetilde{f}$ satisfying C1-5, there exists a targeting strategy f satisfying C1-5 and $f(p^*) = 1$ that produces (weakly) more clicking, where p^* satisfies (MPMC) for f.*

Proof of Lemma 4. Let $\widetilde{p}$ be the price set by the merchant under $\widetilde{f}$. If $\widetilde{f}(\widetilde{p}) = 1$, then let $f = \widetilde{f}$ and $p^* = \widetilde{p}$. If $\int_0^{\widetilde{p}} \widetilde{f}(p) > 0$, then such an f exists by Lemma 3. Otherwise define $f_\epsilon(p) \equiv \widetilde{f}(p) * 1\{p > \widetilde{p}\} + 1\{\widetilde{p} - \epsilon \leq p \leq \widetilde{p}\}$ $\forall \epsilon > 0$. Let p_ϵ be the price set by the merchant under f_ϵ. By (MPMC), $\widetilde{p}\widetilde{f}(\widetilde{p}) \geq \int_{\widetilde{p}}^1 \widetilde{f}(p)dp$. Therefore $\widetilde{p} = \widetilde{p} f_\epsilon(\widetilde{p}) > \int_{\widetilde{p}}^1 f_\epsilon(p)dp$ $\forall \epsilon > 0$. Hence $p_\epsilon < \widetilde{p}$ by (MPMC). Choose an arbitrary small $\epsilon > 0$ such that $\widetilde{p} - \epsilon = (\widetilde{p} - \epsilon) f_\epsilon(\widetilde{p} - \epsilon) > \int_{\widetilde{p}-\epsilon}^1 f_\epsilon(p)dp$. Therefore $p_\epsilon = \widetilde{p} - \epsilon$. Let $f = f_\epsilon$ and $p^* = p_\epsilon$. Note that f adds more slackness to the consumer clicking condition because of equation (4). Therefore if consumers would click on the ad under targeting strategy with the density function $\widetilde{f}$, then consumers would click on the ad under targeting strategy with the density function f. □

Lemma 5. *Given any targeting strategy $\widetilde{f}$ satisfying C1-5, there exists a targeting strategy f satisfying C1-5 and the following functional form condition that produces (weakly) more clicking:*

$$f(p) = \begin{cases} (p^*/p)^2 & \text{if } \widehat{p} \neq p^* \text{ and } p \in (p^*, \widehat{p}] \\ 1 & \text{if } \widehat{p} \neq 1 \text{ and } p \in (\widehat{p}, 1] \end{cases}$$

where $0 \leq p^* \leq \widehat{p} \leq 1$ and p^* satisfies (MPMC) for f.

Proof of Lemma 5. Let $\widetilde{p}$ be the price set by the merchant under $\widetilde{f}$. If $\widetilde{f}$ satisfies the functional form condition, then let $f = \widetilde{f}$ and $p^* = \widetilde{p}$. If no consumers would be induced to click under $\widetilde{f}$, then any targeting strategy is weakly prefereable to $\widetilde{f}$, so choose f that satisfies C1-5 and the functional form condition.

Otherwise $\int_{\widetilde{p}}^1 \widetilde{f}(p)dp < 1 - \widetilde{p}$ and consumers are induced to click under $\widetilde{f}$. By Lemma 4 choose a targeting strategy f_1 that produces weakly more clicking than $\widetilde{f}$ satisfying C1-5 and $f_1(p^*) = 1$, where p^* is the rationally expected price under f_1. Note that $(p^*/p)^2 \leq 1$ $\forall p \geq p^*$.

Because f_1 has no removable, infinite or essential discontinuities and only finite jump discontinuities, I split $(p^*, 1]$ into a countable series of intervals such that: 1) f_1 is either entirely weakly above or entirely weakly below the curve $(p^*/p)^2$ for any given interval, 2) no two weakly above intervals border eachother (I would join these two intervals together to make one interval), and 3) no two weakly below intervals border eachother. Let $\{p_k\}_{k=0,1,\ldots}$ be the sequence of prices at the bounds of these intervals. Note $p_0 = p^*$.

Because $g_1(p) \equiv (p^*/p)^2 1\{p \in (p^*, p_1]\} + f(p) 1\{p \notin (p^*, p_1]\}$ solves the differential equation for $p^* \int_{p^*}^1 g_1(p)dp = p' \int_{p'}^1 g_1(p)dp \, \forall p' \in [p^*, p_1]$, by (MPMC) the first such interval $(p^*, p_1]$ is necessarily weakly above $(p^*/p)^2$. Therefore I choose $\widetilde{p}_1$ satisfying $\int_{p^*}^{p_2} f_1(p)dp = \int_{p^*}^{\widetilde{p}_1} (p^*/p)^2 dp + (p_2 - \widetilde{p}_1)$.

Define $f_2(p) \equiv (p^*/p)^2 1\{p \in (p^*, \widetilde{p}_1]\} + 1\{p \in (\widetilde{p}_1, p_2]\} + f(p) * 1\{p \notin (p^*, p_2]\}$. Note that the merchant would set price p^* under f_2. For prices outside the interval $(p^*, p_2]$, the merchant's demand curve would not change. For prices inside the interval $(p^*, p_2]$, the merchant would have less incentive for raising its price above p^*. Also note that the expected value of clicking, b_c, would increase, so consumers would have more incentive to click. Further note that mass of consumers clicking, Q_c, would remain constant.

Now f_2 on the interval $(p^*, p_3]$ is necessarily weakly above $(p^*/p)^2$, f_2 on the interval $(p_3, p_4]$ is necessarily weakly below $(p^*/p)^2$, and so on. I repeat the same process I used to define f_2 from f_1 to define f_3 from f_2. I iterate over this to get a function that satisfies the functional form conditions. If this takes finite iterations then my terminating function satisfies the functional form conditions. I define this as the targeting strategy f. If this takes infinite iterations then I use the limit of the subsequence of the functions (which must exist because my choice of a function is bounded). Let me call this function $\widehat{f}$. $\widehat{f}$ would not have any essential or infinite discontinuities because $\widehat{f}$ is bounded and the slope of $\widehat{f}$ is bounded from below. I eliminate all the removable discontinuities from $\widehat{f}$ and revalue my jump discontinuities by C4 and C5. I call this function f, which satisfies the functional form conditions and C1-5. □

Proof of Proposition 1. This follows directly from Lemmas 3, 4, and 5. □

Lemma 6. *If $b_1 < b$ and there exists a targeting strategy $\widetilde{f}$ satisfying C1-5 that would induce consumers to click, then the advertising platform would choose a targeting strategy such that $b_c = b$.*

Proof of Lemma 6. By way of contradiction suppose not. Suppose $b > b_1$ and there exists a targeting strategy $\widetilde{f}$ that would induce consumers to click satisfying C1-5. Suppose there exists a click maximizing targeting strategy f satisfying C1-5 and $b_c = \int_{p^*}^1 (p - p^*) f(p)dp \neq b$ where p^* be the price set by the merchant under f. If $b_c < b$ then f would not induce any consumers to click on the ad, so $b_c > b$. By Lemma 2, the advertisement platform shows the ad to some but not all of the consumers so $0 < \int_0^1 f(p)dp < 1$. Because $\int_0^1 f(p)dp < 1$ and f has only finite discontinuities, choose a price $p' \in (0,1)$ such that $f(p') < 1$ and f is continuous at p'. Define $f_\epsilon(p) \equiv f(p) * 1\{p \in [0, p' - \epsilon]\} + 1\{p \in (p' - \epsilon, p' + \epsilon]\} + f(p) * 1\{p \in (p' + \epsilon, 1]\}$ $\forall \epsilon \in (max\{p', 1 - p, \}, 0)$. Let p_ϵ be the price set by the merchant under f_ϵ. Because f is continuous at p', choose an $\epsilon > 0$ such that $\int_{p_\epsilon}^1 (p - p_\epsilon) f_\epsilon(p)dp > b$. Consumers would still be induced to click under f_ϵ by (CCC) and $\int_0^1 f_\epsilon(p)dp > \int_0^1 f(p)dp$. Thus the advertisement platform strictly prefers the feasible targeting strategy f_ϵ to f. □

Proof of Proposition 2. Proof by parts:

Part 1. When $b \leq b_1$: $f(p) = 1\{p \in [0,1]\}$.

Define $b_1 \equiv \int_{p^m}^1 (p - p^m)dp = 1/8$, where $p^m = 1/2$ is the price set by the merchant facing the demand curve $(1-p)1\{p \in [0,1]\}$. By Lemma 1, if $b \leq b_1$, then the unique optimal targeting strategy is $f(p) = 1\{p \in [0,1]\}$.

Part 2. The search costs $b > b_1$ such that there exists a targeting strategy $\widetilde{f}$ satisfying C1-5 that would induce consumers to click form a contiguous interval with an infimum of b_1.

Define $b_s \equiv b_1/[1 - p^m]$, where $p^m = 1/2$ is the price set by the merchant facing the demand curve $(1-p)1\{p \in [0,1]\}$. By (CCC), if the search cost b were less than or equal to b_s, then the consumers would click under the targeting strategy $f(p) = 1\{p \in [p^m, 1]\}$. Because $b_s > b_1$, there exists search costs $b > b_1$ for which the targeting strategy $m = f$ does not satisfy (CCC) and at least one other targeting strategy $\widetilde{f}$ satisfies (CCC).

If the ad platform has a strategy that can induce some consumers to click for search cost b', then it can use the same strategy to induce some consumers to click for search cost $b'' < b'$. Therefore, by Lemma 2, the search costs $b > b_1$ such that there exists a targeting strategy $\widetilde{f}$ satisfying C1-5 that would induce consumers to click form a contiguous interval with an infimum of b_1.

Part 3. *Given any* $\underline{p} \in (0, p^*(\underline{p}, \widehat{p}))$ *and any* $\widehat{p} \in [p^*(\underline{p}, \widehat{p}), p_1)$ *there exists a* $\underline{p}' \in (0, \underline{p})$ *and a* $\widehat{p}' \in (\widehat{p}, p_1)$ *satisfying* $\overline{F}(\underline{p}, \widehat{p}) = \overline{F}(\underline{p}', \widehat{p}')$ *and* $b_c(\underline{p}', \widehat{p}') > b_c(\underline{p}, \widehat{p})$.

Define $f_{(\underline{p},\widehat{p})}(p) \equiv 1\{p \in [\underline{p}, p^*(\underline{p}, \widehat{p})]\} + (p^*/p)^2 1\{p \in (p^*(\underline{p}, \widehat{p}), \widehat{p}]\} + 1\{p \in (\widehat{p}, 1]\}$, where $p^*(\underline{p}, \widehat{p})$ is the rationally expected price under $f_{(\underline{p},\widehat{p})}$. Define $\overline{F}(\underline{p}, \widehat{p}) \equiv \int_0^1 f_{(\underline{p},\widehat{p})}(p)dp$. Note that $f_{(\underline{p},\widehat{p})}$ satisfies C1-5 for any $\underline{p} \in (0, p^*(\underline{p}, \widehat{p}))$ and any $\widehat{p} \in [p^*(\underline{p}, \widehat{p}), p_1)$.

I will begin by analyzing the case where $\underline{p} < p^*(\underline{p}, \widehat{p})$. By (MPMC) when $\widehat{p} > 1/2 = p^m$, I have $p^* = \int_{p^*}^{\widehat{p}} [(p^*/p)^2]dp + 1 - \widehat{p}$, therefore $p^*(\underline{p}, \widehat{p}) = \sqrt{\widehat{p}(1-\widehat{p})}$. Note that changing $\underline{p}$ does not change $p^*(\underline{p}, \widehat{p})$, because the merchant would not choose to sell to those consumers anyway, so I write $p^*(\widehat{p}) = \sqrt{\widehat{p}(1-\widehat{p})}$. Also note that if $\widehat{p} \leq 1/2$ then I would not have a targeting strategy in the form given by Proposition 1, because the merchant would set $p^* = 1/2$. Further note that when $\underline{p} < p^*(\underline{p}, \widehat{p})$, I have $\overline{F}(\underline{p}, \widehat{p}) = 2p^*(\underline{p}, \widehat{p}) - \underline{p} = 2\sqrt{\widehat{p}(1-\widehat{p})} - \underline{p}$.

The advertisement platform can increase the expected benefit b_c of clicking on the ad in two ways: through increasing $\underline{p}$ and through increasing $\widehat{p}$. Doing so decreases the mass $\overline{F}(\underline{p}, \widehat{p})$ of consumers clicking. The decrease in the mass from increasing one choice variable can be offset by the increase in the mass from decreasing the other choice variable. The effect of changing the expected benefit b_c of clicking through increasing $\underline{p}$ (and holding $\widehat{p}$ constant) is given in equation (5) and through increasing $\widehat{p}$ (and holding $\underline{p}$ constant) is given in equation (6).

$$\frac{\partial b_c}{\partial \overline{F}(\underline{p}, \widehat{p})}(\underline{p}, \widehat{p})\bigg|_{through\, \underline{p}} = \frac{\partial b_c}{\partial \underline{p}}(\underline{p}, \widehat{p}) \Big/ \frac{\partial \overline{F}(\underline{p}, \widehat{p})}{\partial \underline{p}}(\underline{p}, \widehat{p}) = -\frac{CS(\underline{p}, \widehat{p})}{\overline{F}(\underline{p}, \widehat{p})^2} \tag{5}$$

where $CS(\underline{p}, \widehat{p}) \equiv \int_{p^*(\widehat{p})}^1 (p - p^*(\widehat{p}))f_{(\underline{p},\widehat{p})}(p)dp = \frac{\widehat{p}(1-\widehat{p})}{2} \ln(\frac{\widehat{p}}{1-\widehat{p}}) + \frac{(1-\widehat{p})^2}{2}$

Note: $b_c = CS(\underline{p}, \widehat{p})/\overline{F}(\underline{p}, \widehat{p})$

$$\begin{aligned} \frac{\partial b_c}{\partial \overline{F}(\underline{p}, \widehat{p})}(\underline{p}, \widehat{p})\bigg|_{through\, \widehat{p}} &= \frac{\partial b_c}{\partial \widehat{p}}(\underline{p}, \widehat{p}) \Big/ \frac{\partial \overline{F}(\underline{p}, \widehat{p})}{\partial \widehat{p}}(\underline{p}, \widehat{p}) \\ &= \frac{\partial CS(\underline{p}, \widehat{p})/\partial \widehat{p}}{\overline{F}(\underline{p}, \widehat{p}) * \partial \overline{F}(\underline{p}, \widehat{p})/\partial \widehat{p}} - \frac{CS(\underline{p}, \widehat{p})}{\overline{F}(\underline{p}, \widehat{p})^2} \end{aligned} \tag{6}$$

Here, $CS(\underline{p},\widehat{p})/\overline{F}(\underline{p},\widehat{p})^2$ is the increase in b_c through decreasing the mass $\overline{F}(\underline{p},\widehat{p})$ of consumers clicking by increasing $\underline{p}$. Because $p^*(\widehat{p})$ and $f_{(\underline{p},\widehat{p})}(p)$ above $p^*(\widehat{p})$ does not depend on $\underline{p}$, changing $\underline{p}$ does not affect the Consumer Surplus $CS[\underline{p},\widehat{p}]$, so changing $\underline{p}$ only affects b_c through decreasing the mass $\overline{F}(\underline{p},\widehat{p})$. This affect is $CS(\underline{p},\widehat{p})/\overline{F}(\underline{p},\widehat{p})^2$.

Yet, changing $\widehat{p}$ changes p^*, so changing $\widehat{p}$ changes b_c through changing $\overline{F}(\underline{p},\widehat{p})$ and $CS(\underline{p},\widehat{p})$. Equation (7) decomposes $\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}$.

$$\frac{\partial CS(\underline{p},\widehat{p})}{\partial\widehat{p}} = PE(\widehat{p}) - Y(\widehat{p}) - Z(\widehat{p}) \tag{7}$$

$$\text{where } PE(\widehat{p}) \equiv -(1-\widehat{p})\tfrac{\partial p^*(\widehat{p})}{\partial\widehat{p}} = -\tfrac{(1-\widehat{p})(1-2\widehat{p})}{2\sqrt{\widehat{p}(1-\widehat{p})}} > 0$$

$$\begin{aligned}\text{where } Y(\widehat{p}) &\equiv -\int_{p^*(\widehat{p})}^{\widehat{p}} \frac{\partial}{\partial\widehat{p}}[(p-p^*(\widehat{p}))\frac{(p^*(\widehat{p}))^2}{p^2}]dp \\ &= -\frac{3}{2}(1-2\widehat{p})(\frac{1}{3}\ln\frac{\widehat{p}}{1-\widehat{p}} + \sqrt{\frac{1-\widehat{p}}{\widehat{p}}} - 1)\end{aligned}$$

$$\text{and where } Z(\widehat{p}) \equiv (\widehat{p}-p^*(\widehat{p}))(1-\tfrac{(p^*(\widehat{p}))^2}{\widehat{p}^2}) = -(1-2\widehat{p})(1-\sqrt{\tfrac{1-\widehat{p}}{\widehat{p}}}) > 0$$

Here $PE(\widehat{p})$ is the *price effect* on all consumers with reservation prices above $\widehat{p}$. It reflects how much each consumer with a reservation price above $\widehat{p}$ will benefit from the lowering of the price set by the merchant. $Y(\widehat{p})$ is the *infra-marginal consumer loss effect*. By increasing $\widehat{p}$, the ad platform is inducing the merchant to choose a lower price $p^*(\widehat{p})$. By lowering $p^*(\widehat{p})$, the constant $(p^*(\widehat{p}))^2$ is lower. This in turn lowers the ad platforms choice of $f_{(\underline{p},\widehat{p})}$ between $p^*(\widehat{p})$ and $\widehat{p}$. $Z(\widehat{p})$ is the *marginal consumer loss effect*. By increasing $\widehat{p}$, the advertisement platform is not advertising to some consumers with reservation prices $r=\widehat{p}$. $Z(\widehat{p})$ captures the effect of the loss of the advertising to these consumers on the consumer surplus.

Note that this shows that $\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}$ is independant of $\underline{p}$.

As $\widehat{p}$ approaches $p^m = 1/2$: the price effect $PE(\widehat{p})$, the infra-marginal consumer loss effect $Y(\widehat{p})$ and the marginal consumer loss effect $Z(\widehat{p})$ converge to zero. And $PE'(\widehat{p})$ converges to 1, while $Y'(\widehat{p})$ and $Z'(\widehat{p})$ converge to zero. Therefore for small values of $\widehat{p}$, $\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}$ is strictly greater than zero.

As $\widehat{p}$ approaches 1: the price effect $PE(\widehat{p})$ converges to zero, the infra-marginal consumer loss effect $Y(\widehat{p})$ goes to infinity, and the marginal consumer loss effect $Z(\widehat{p})$ goes to 1. Thus $\partial CS(\underline{p}, \widehat{p})/\partial\widehat{p}$ goes to negative infinity.

Because $PE(\widehat{p})$, $Y(\widehat{p})$ and $Z(\widehat{p})$ are continuous with respect to changes in $\widehat{p}$, $\partial CS(\underline{p}, \widehat{p})/\partial\widehat{p}$ is continuous with respect to changes in $\widehat{p}$. Therefore by the Intermediate Value Theorem, there exists at least one $p_1 \in (1/2, 1)$ such that $\partial CS(\underline{p}, p_1)/\partial\widehat{p} = 0$. Setting (7) equal to zero, I have $p_1 = e/(1+e) \approx 0.731$.

Therefore when $\underline{p} < p^*(\underline{p}, \widehat{p})$ and $\widehat{p}$ is less than $p_1 = e/(1+e)$, the ad platform can increase b_c, while holding the mass $\overline{F}$ of consumers clicking constant by increasing $\widehat{p}$ while decreasing $\underline{p}$.

<u>Part 4.</u> Given any $\underline{p} = p^*(\underline{p}, \widehat{p})$ and any $\widehat{p} \in [p^*(\underline{p}, \widehat{p}), p_1)$ there exists a $\underline{p}' \in (0, \underline{p})$ and a $\widehat{p}' \in (\widehat{p}, p_1)$ satisfying $\overline{F}(\underline{p}, \widehat{p}) = \overline{F}(\underline{p}', \widehat{p}')$ and $b_c(\underline{p}', \widehat{p}') > b_c(\underline{p}, \widehat{p})$.

Define $f_{(\underline{p},\widehat{p})}(p)$, $p^*(\underline{p}, \widehat{p})$, and $\overline{F}(\underline{p}, \widehat{p})$ as in Part 3.

This uses the argument as Part 3, with a caveat: I need to show that the ad platform would never set $\underline{p}$ above the price $p^*(\underline{p}, \widehat{p})$ when the FOC given by (2) does not hold. Obviously if the FOC holds then it is true by taking the limit of the argument given in Part 3.

If $p^* < \int_{p^*}^1 f_{(\underline{p},\widehat{p})}(p)dp$, then the merchant would set a price $p^* > \underline{p}$. Therefore consider when $\underline{p} = p^*(\underline{p}, \widehat{p})$ and $\underline{p} > \int_{\underline{p}}^1 f_{(\underline{p},\widehat{p})}(p)dp$. Choose an $\epsilon > 0$ small enough so $\underline{p} - \epsilon = p^*(\underline{p} - \epsilon, \widehat{p})$ and $(\underline{p} - \epsilon) > \int_{\underline{p}-\epsilon}^1 f(p)dp$. Targeting strategy $f_{(\underline{p}-\epsilon,\widehat{p})}$ would produce a higher b_c than $f_{(\underline{p},\widehat{p})}$ and $\overline{F}(\underline{p} - \epsilon, \widehat{p}) > \overline{F}(\underline{p}, \widehat{p})$.

Therefore when $\underline{p} = p^*(\underline{p}, \widehat{p})$, the ad platform can increase b_c, while holding the mass $\overline{F}$ of consumers clicking constand by choosing a smaller $\underline{p}$ and a larger $\widehat{p}$ to compensate for the $\overline{F}$ gained due to a smaller $\underline{p}$.

<u>Part 5.</u> When $b_1 < b \leq b_2$: $f(p) = 1\{p \in [0, p^*]\} + (p^*/p)^2 1\{p \in (p^*, \widehat{p}]\} + 1\{p \in (\widehat{p}, 1]\}$ where $\widehat{p} \in (p^m, p_1]$

Define $f_{(\underline{p},\widehat{p})}(p)$, $p^*(\widehat{p})$, and $\overline{F}(\underline{p}, \widehat{p})$ as in Part 3.

$$
\begin{aligned}
\text{Define } b_2 &\equiv \int_{p^*(p_1)}^1 (p - p^*(p_1)) f_{(0,p_1)}(p)dp/\overline{F}(0, p_1) \\
&= \frac{1}{4\sqrt{e}} \approx 0.152
\end{aligned}
$$

Note: $p^*(p_1) = \frac{\sqrt{e}}{1+e}$

By (CCC), if the search cost $b \leq b_2$, then the consumers would click under the targeting strategy $f_{(0,p_1)}$.

By Parts 3 and 4, if the search cost b satisfies $b_1 < b_2 < b2$, then the click maximizing targeting strategy that gives just enough expected benefit b_c to get consumers to click would be for $\underline{p}$ to be as low as possible; in this case, $\underline{p} = 0$.

<u>Part 6.</u> *Given any* $\underline{p} \in (0, p^*(\underline{p}, \widehat{p}))$ *and any* $\widehat{p} \in (p_1, p_2]$ *there exists a* $\underline{p}' \in (\underline{p}, p^*(\underline{p}', \widehat{p}'))$ *and a* $\widehat{p}' \in (p_1, \widehat{p})$ *satisfying* $\overline{F}(\underline{p}, \widehat{p}) = \overline{F}(\underline{p}', \widehat{p}')$ *and* $b_c(\underline{p}', \widehat{p}') > b_c(\underline{p}, \widehat{p})$.

This follows directly from the argument given in Part 3. For $\widehat{p} > p_1$, $\partial CS(\underline{p}, \widehat{p})/\partial \widehat{p} < 0$.

<u>Part 7.</u> *When* $b_2 < b \leq b_3$*:* $f(p) = 1\{p \in [\underline{p}, p^*]\} + (p^*/p)^2 1\{p \in (p^*, p_1]\} + 1\{p \in (p_1, 1]\}$ *where* $\underline{p} \in (0, p^*]$.

Define $f_{(\underline{p},\widehat{p})}(p)$, $p^*(\widehat{p})$, and $\overline{F}(\underline{p}, \widehat{p})$ as in Part 3.

$$\text{Define } b_3 \equiv \int_{p^*(p_1)}^{1} (p - p^*(p_1)) f_{(0,p_1)}(p) dp / \overline{F}(0, p_1)$$
$$= \frac{1}{2\sqrt{e}} \approx 0.303$$

Note: $p^*(p_1) = \frac{\sqrt{e}}{1+e}$

By (CCC), if the search cost $b \leq b_3$, then the consumers would click under the targeting strategy $f_{(p^*(p_1),p_1)}$.

By Part 6, if the search cost b satisfies $b_2 \leq b \leq b_3$, the click maximizing targeting strategy would have $\widehat{p} = p_1$. The price cutoff $\underline{p}$ would be just low enough to give the expected benefit b_c equal to the search cost b.

<u>Part 8.</u> *When* $b_3 < b \leq b_4$*:* $f(p) = (p^*/p)^2 1\{p \in [p^*, \widehat{p}]\} + 1\{p \in (\widehat{p}, 1]\}$ *where* $\widehat{p} \in (p_1, p_2]$.

Define $f_{(\underline{p},\widehat{p})}(p)$, $p^*(\widehat{p})$, and $\overline{F}(\underline{p}, \widehat{p})$ as in Part 3.

After increasing $\widehat{p}$ to p_1 and increasing $\underline{p}$ to p^*, then the expected benefit of clicking, b_c , still might be large enough to induce the consumer to click on the ad. As long as the equation (6) is negative, the ad platform can still increase the expected benefit from clicking on the ad by increasing $\widehat{p}$. Increasing $\widehat{p}$ further still increases the expected benefit from clicking on the ad through decreasing the mass of consumers clicking on the ad. When $\left|\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}\right| > \left|\partial\overline{F}(\underline{p},\widehat{p})/\partial\widehat{p} * CS(\underline{p},\widehat{p})/\overline{F}(\underline{p},\widehat{p})\right|$, then the advertisement platform can still increase the expected benefit of clicking on the ad by increasing $\widehat{p}$.

I have established that $\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}$ is zero when $\widehat{p} = p_1$ and negative infinity as $\widehat{p} \to 1$. Because of the continuity of $\partial CS(\underline{p},\widehat{p})/\partial\widehat{p}$ and $\partial\overline{F}(\underline{p},\widehat{p})/\partial\widehat{p} * CS(\underline{p},\widehat{p})/\overline{F}(\underline{p},\widehat{p})$, there must exist a $p_2 > p_1$ such that $\partial CS(\underline{p},p_2)/\partial\widehat{p} = \partial\overline{F}(\underline{p},p_2)/\partial\widehat{p} * CS(\underline{p},p_2)/\overline{F}(\underline{p},p_2)$. Setting $\underline{p} = p^* = \sqrt{\widehat{p}(1-\widehat{p})}$ and setting equation (6) equal to zero, I find that p_2 solves $(1+p_2)/p_2 = \ln[p_2/(1-p_2)]$. Therefore $p_2 \approx .893$.

$$\begin{aligned}
\text{Define } b_4 &\equiv \int_{p^*(p_2)}^{1} (p - p^*(p_2)) f_{(p^*(p_2),p_2)}(p)dp / \overline{F}(p^*(p_2),p_2) \\
&= \frac{\sqrt{p_2(1-p_2)}}{2}\Big(\ln\frac{p_2}{1-p_2} + \frac{1-p_2}{p_2}\Big) \\
&\approx 0.346
\end{aligned}$$

By (CCC), if the search cost $b \leq b_4$, then the consumers would click under the targeting strategy $f_{(p^*(p_2),p_2)}$.

By the argument in Part 3, if the search cost b satisfies $b_3 \leq b \leq b_4$, the click maximizing targeting strategy would want to lower $\widehat{p}$ to increase the expected benefit b_c to b.

Part 9. *When $b > b_4$: no consumer would ever click on the ad so any m is optimal.*

If $b > b_4$ then it is impossible for the advertising platform to induce the consumers to click on the ad. If there were a strategy f'' then there would be a strategy of the form given in Proposition 1 that could induce consumers to click on the ad. But my comparative static results in Part 2 rule out this possibility. □

Lemma 7. *When $b \leq b_5 \equiv 1/4$, then a merchant-profit-maximizing targeting strategy is $f(p) = 1\{p \in [p^m, 1]\}$.*

Proof of Lemma 7. Suppose not. Let $\widetilde{f}$ satisfying C1-5 maximize the merchant's profits. Let $\widetilde{p}$ be the price set by the merchant under $\widetilde{f}$.

Under the demand function $1 - p$, the merchant would choose to set its price as $p^m = 1/2$. Therefore by the Weak Axiom of Profit Maximization: $p^m(1 - p^m) \geq \widetilde{p}(1 - \widetilde{p})$. I have that $1 - \widetilde{p} \geq \int_{\widetilde{p}}^1 \widetilde{f}(p)dp$, by C1. Thus $p^m(1 - p^m) \geq \widetilde{p} \int_{\widetilde{p}}^1 \widetilde{f}(p)dp$.

Note that consumers would click under the targeting strategy $f(p) = 1\{p \in [p^m, 1]\}$ by (CCC). And that this gives the merchant the profit $p^m(1 - p^m) = b_5 = 1/4$. □

Proof of Proposition 3. Define $b_5 \equiv \int_{p^m}^1 (p-p^m)dp/(1-p^m) = 1/8$, where p^m is the price set by the merchant facing the demand curve $1 - p$. And define b_4 as in Part 8 of the proof of Proposition 2.

When $b \leq b_5$, consumers would click under the targeting strategy $f_0(p) \equiv 1\{p \in [p^m, 1]\}$ by (CCC). This maximizes the merchants profit by Lemma 7.

When $b_5 \leq b \leq b_4$, consumers would not click under f_0, so the ad platform has to choose a targeting strategy that commits the merchant to a lower price to encourage them to click. By Lemma 6, the ad platform would do so to make $b_c = b$. By Proposition 1, the ad platform would do so with a targeting strategy of the form (a) or (b) of Proposition 1. Raising $\underline{p}$ would not affect the merchant's profit as long as $\underline{p} < p^*$, so the ad platform would set $\underline{p}$ equal to the rationally expected price $p^* = \widehat{p}(1 - \widehat{p})$ of the merchant. But doing so would not be enough (because f_0 does not induce consumers to click), so the ad platform would raise $\widehat{p}$ above p^m. Because $b \leq b_4$, it is possible to induce consumers to click with a high enough $\widehat{p}$.

When $b > b_4$, by the argument in Part 9 of the proof of Proposition 2, it is impossible for the ad platform to induce consumers to click. □

B Functional Form Conditions

C1 $f(p) \in [0, 1]$ for all $p \in [0, 1]$.

C2 f is not discontinuous at an infinite number of points.

C3 for all $p \in [0, 1]$, the right-hand and left-hand limits of $f(p)$ exists and are finite.

C4 $\lim_{p \to \underline{p}^+} f(p) = f(\underline{p})$ where $\underline{p}$ is the smallest $p \in [0, 1]$ such that $f(p) \neq 0$.

C5 $\lim_{p \to p'^-} f(p) = f(p')$ for all $p' \neq \underline{p}$.

C1 prevents the advertising platform from showing the ad to more consumers than those who exist. C2 is necessary so we can split f into a finite number of continuous intervals. C3 ensures that f is integrable and that there are no essential discontinuities.[24] C4 and C5 prevent removable discontinuities and restrict the jump discontinuities to be of the most convenient direction for my mathematical proofs. Note that C4 and C5 are not that restrictive because they do not restrict the mass of consumers shown an ad along any interval.

[24]C2 and C3 are not the same as f being measurable. Consider the function f' which is one for all rational numbers and zero for all irrational numbers. f' is measurable, but violates both C2 and C3